Homework 10:

1) Given a short exat sequence $0 \to F_1 \to F_0 \to A \to 0$ of abelian groups, show that the functor $hom(, B)$ from abelian groups to itself is a left-exact functor, i.e., the sequence

$$0 \to hom(A, B) \xrightarrow{f_0^*} hom(F_0, B) \xrightarrow{f_1^*} hom(F_1, B) \to 0$$

is exact except on the right end.

2) Compute $H_*(K)$, where K is the Klein bottle. Now compute $H_*(K; \mathbb{Z}/2^n)$ (i) by a direct argument, and (ii) using universal coefficients.

3) Work over $R = \mathbb{Z}$ and fix distinct primes p and q. Compute (i) $Ext(\mathbb{Z}, \mathbb{Z}/p^n)$; (ii) $Ext(Z/p^n, \mathbb{Z})$; (iii) $Ext(\mathbb{Z}/p^n, \mathbb{Z}/q^m)$; (iv) $Ext(\mathbb{Z}/p^n, \mathbb{Z}/p^m)$.

4) If $H_*(X)$ is finitely generated, show that $\chi(X) = \sum(-1)^i \dim H_i(X; R)$ for any field R.

5) View $\mathbb{R}P^2$ as the quotient of the closed unit disk D^2 with its opposite boundary points identified. We can construct a map $q : \mathbb{R}P^2 \to S^2$ by viewing S^2 as the quotient of the closed unit disk D^2 with all of its boundary points identified. Show that for $R = M = \mathbb{Z}_2$, the universal coefficients exact sequences for homology cannot be split compatibly with q.

6) Hatcher, Problem 11, page 205.

7) Suppose X and Y are contractible topological spaces. Show that the chain complex $C_*(X) \otimes C_*(Y)$ is acyclic away from degree 0.

8) Hatcher, Problem 4, page 205.

9) (Optional) Give a short exact sequence relating $H^*(X; M)$ and $H^*(X)$.