Homework 12:

1) Verify that Alexander-Whitney diagonal approximation is a chain map.

2) Prove that the cohomology groups of a compact, simply connected, orientable 4-manifold M are completely determined by the second Betti number $b_2(M) = dim_{\mathbb{Q}} H^2(M;\mathbb{Q})$. Prove that the cohomology ring is determined by the cup-product pairing $H^2 \times H^2 \to \mathbb{Z}$.

3) Show that a topological group which is also a manifold, is always orientable.

4) Show that every covering space of an orientable manifold is orientable.

5) Show that if a manifold is orientable ($\mathbb{Z}$-orientable), then it is R-orientable for any commutative ring R.

6) Hatcher, page 258, pb. 6.

7) Hatcher, page 257, pb. 7.