Homework 13:

1) Show that the Euler characteristic of a compact, orientable, odd-dimensional manifold is zero.

2) Using Poincaré duality, give another computation of the cohomology ring of $H^*(\mathbb{C}P^n; \mathbb{Z})$.

3) For which even dimensions $2n$ is it true that the Euler characteristic of a compact, connected, orientable $2n$-manifold is necessarily even?

4) Hatcher page 259, pb. 24

5) Hatcher page 259, pb. 25

6) Show that for $U \subset \mathbb{R}^3$ open, $H_1(U)$ is torsion-free. (Note that this is, in general, false for $\mathbb{R}^n$, $n > 3$.)

7) Let M be a closed, connected, orientable n-manifold and let $f : S^n \to M$ be map of nonzero degree. (Recall this means that $f_*[S^n] = k[M] \in H_n(M)$ for some non-zero k). Then show that $H_*(M; \mathbb{Q}) \cong H_*(S^n; \mathbb{Q})$. If the degree is ± 1, then show that $H_*(M; \mathbb{Z}) \cong H_*(S^n, \mathbb{Z})$.

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You did it! This was the last problem set.